LA-UR-19-28868
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Title: Solving discontinuous problems with pseudospectral methods

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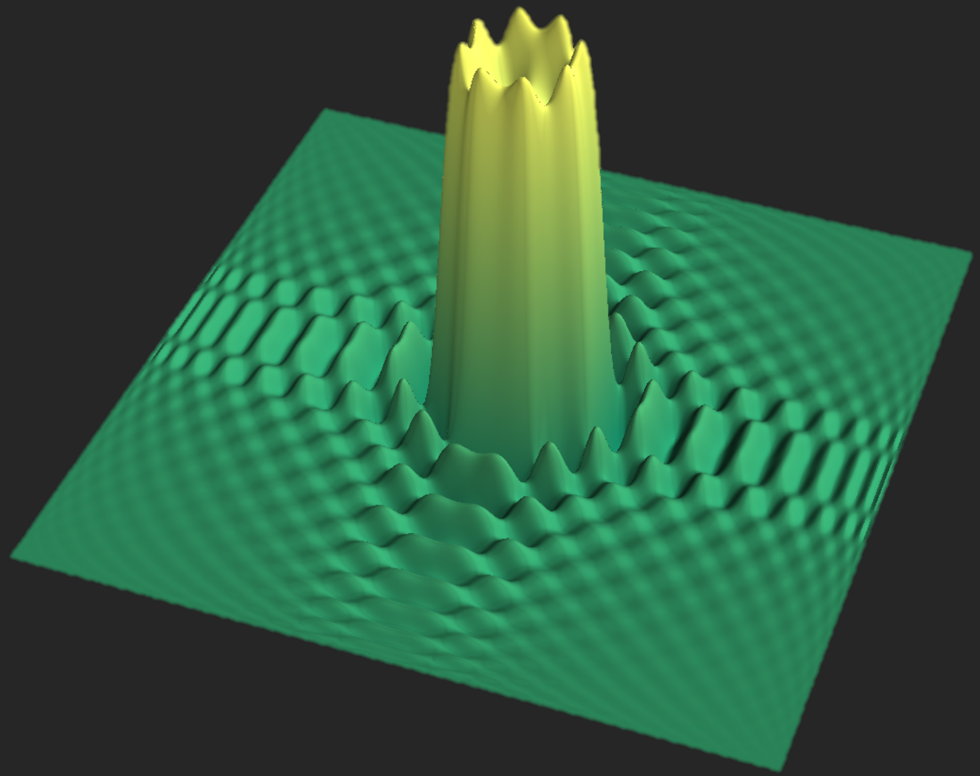
Intended for: Presentation at my university

Issued: 2019-09-03



Solving discontinuous problems with **pseudospectral methods**

JOANNA PIOTROWSKA, JONAH MILLER

Why investigate numerical methods?

... IF THERE ALREADY EXIST WIDELY APPLIED SCHEMES?

PSEUDOSPECTRAL METHODS

- Projection onto a set of basis functions:

$$(I_N f)(x) = \sum_{n=0}^{N} \phi_n T_n(x)$$

where:

$$\phi_n = \frac{1}{\gamma_n} \sum_{j=0}^{N} f(x_j) T_n(x_j) w_j,$$

$$\gamma_n = \sum_{j=0}^{N} {T_n}^2 (x_j) w_j$$

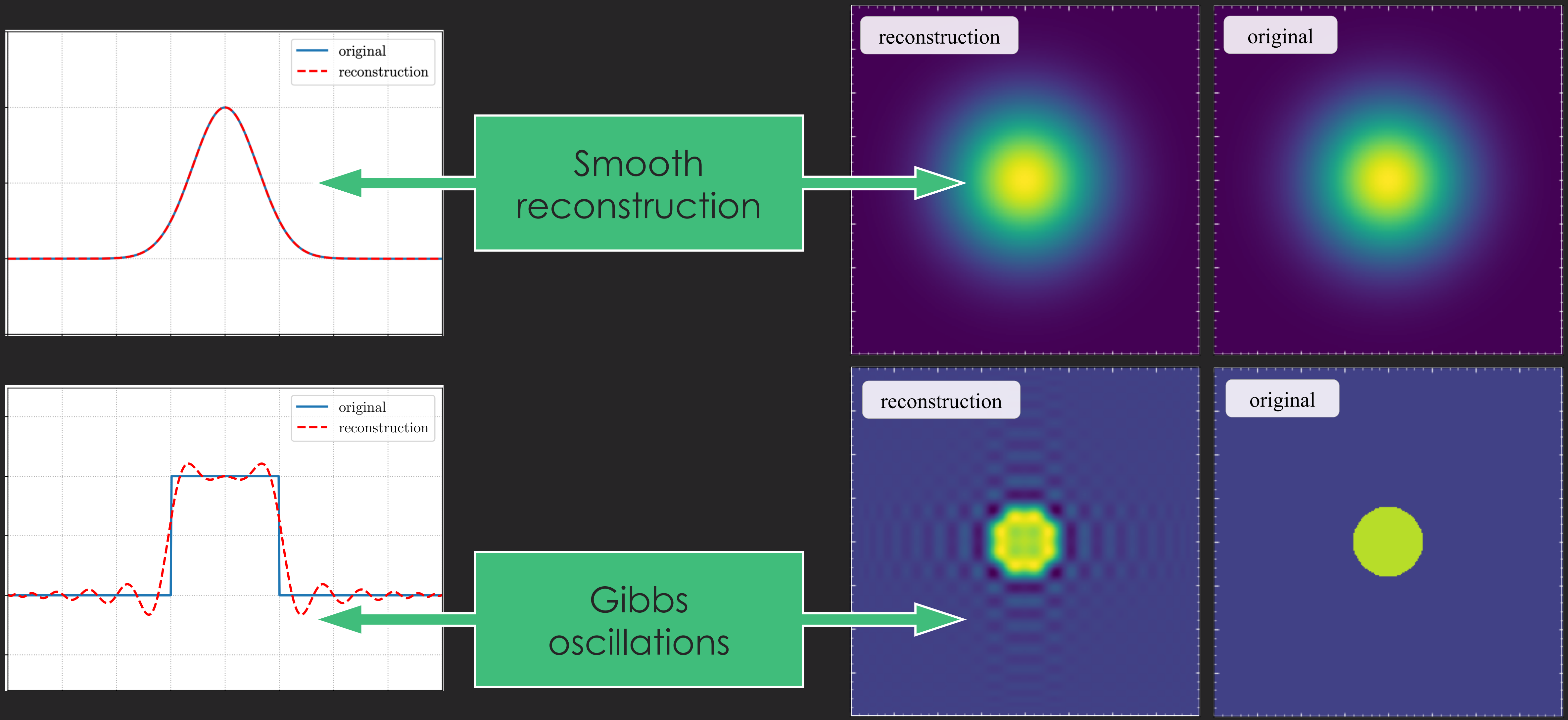
ADVANTAGES

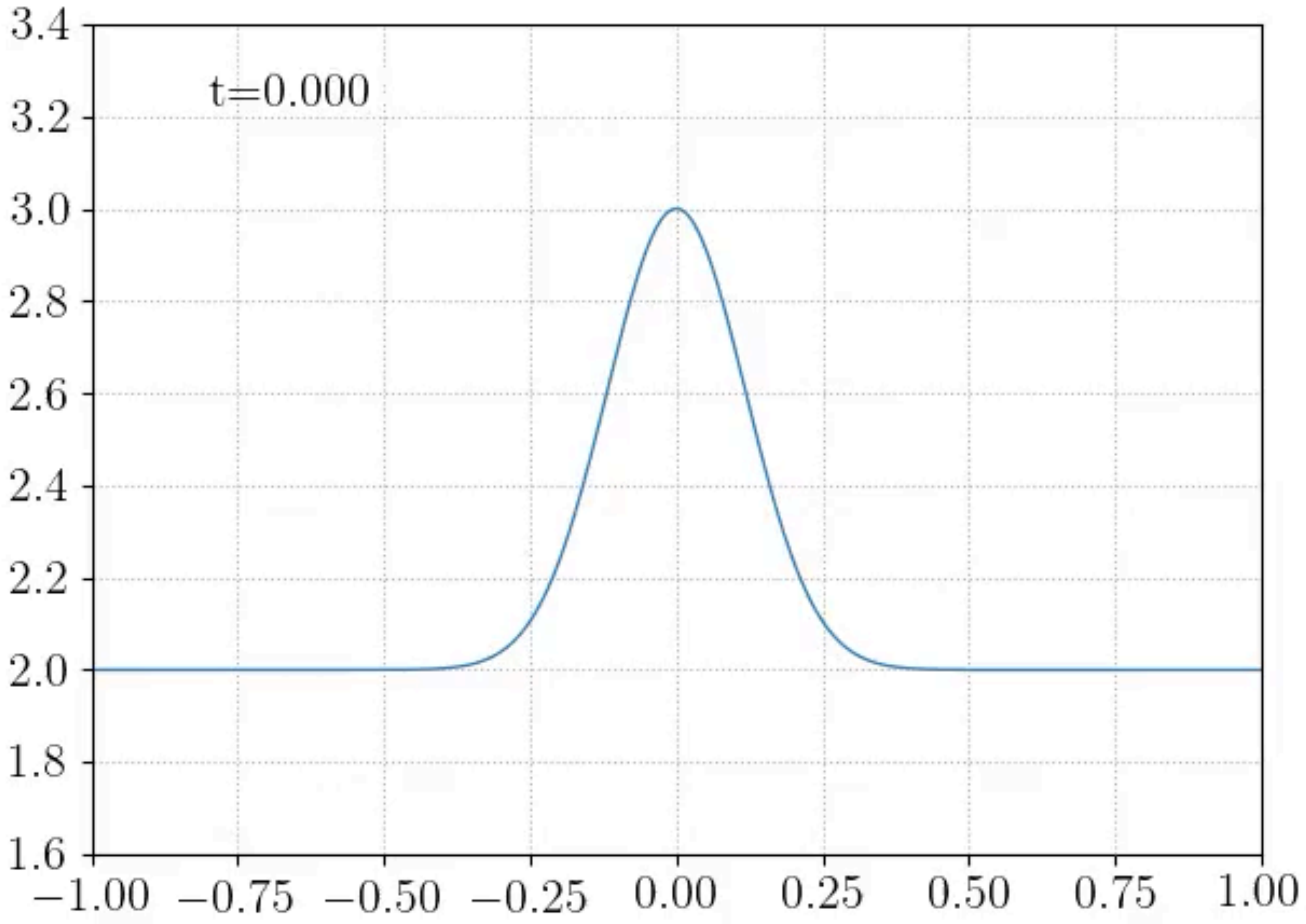
Exponential error decay for <u>smooth solutions</u>

ISSUES

Gibbs phenomenon in discontinuous solutions

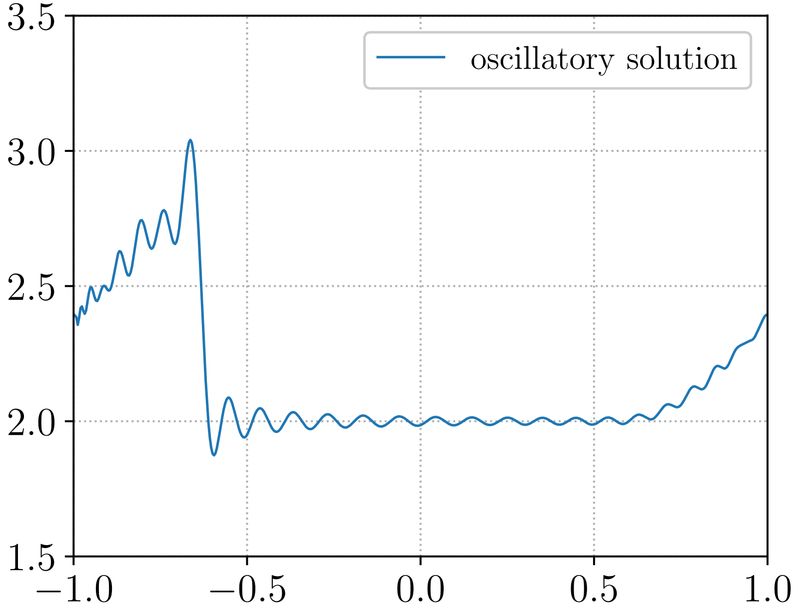
GIBBS PHENOMENON



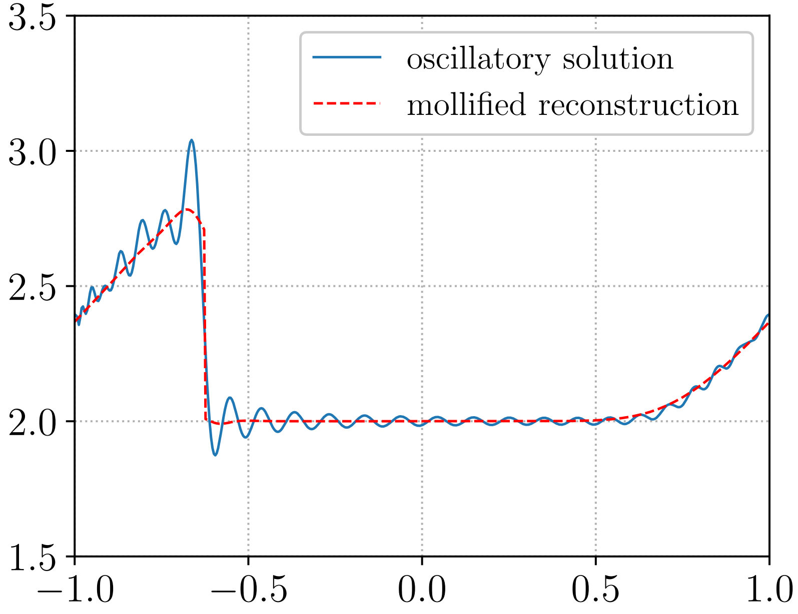

t=0.000
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

A system involving **shocks can be solved with pseudospectral methods**

Evolve the solution in time
at the collocation points

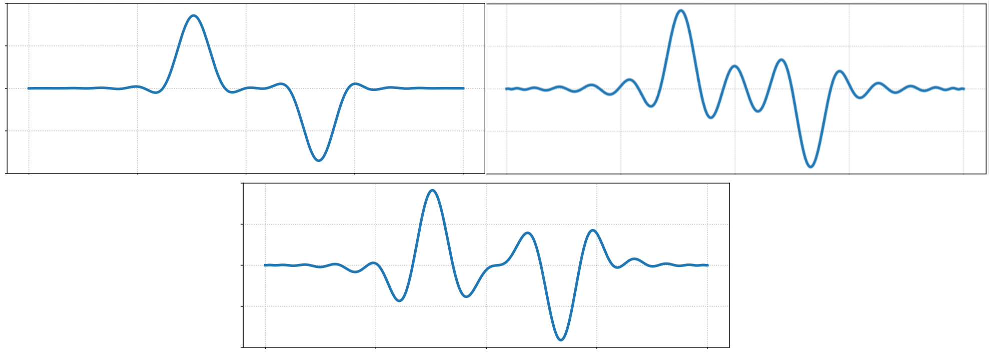


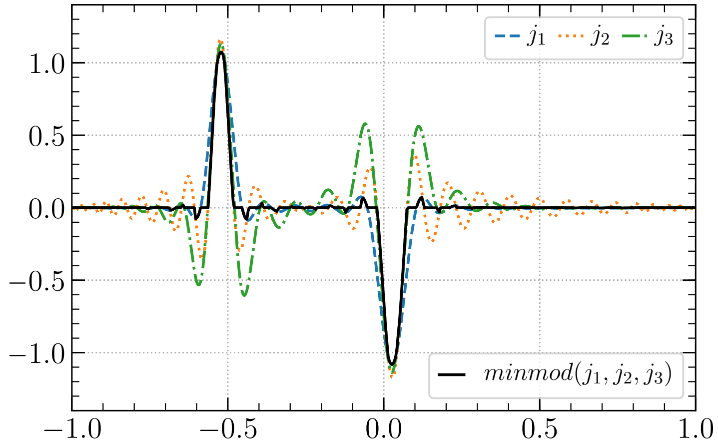
Remove Gibbs oscillations
in post-processing



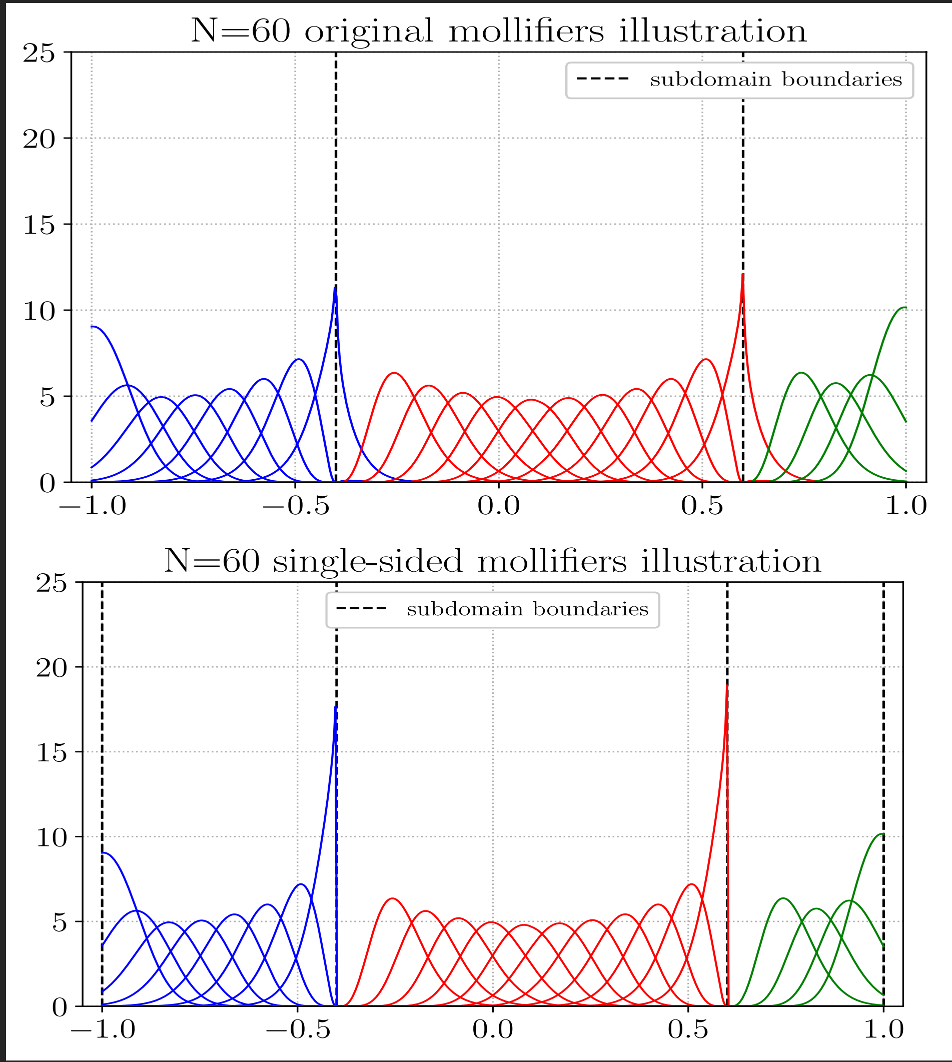
EDGE DETECTION

$$\frac{\sqrt{1-x^2}}{N}\sum_{k=1}^{N}\mu\left(\frac{k}{N}\right)\hat{f}(k)T'_k(x)\longrightarrow [f](x)$$





MOLLIFICATION

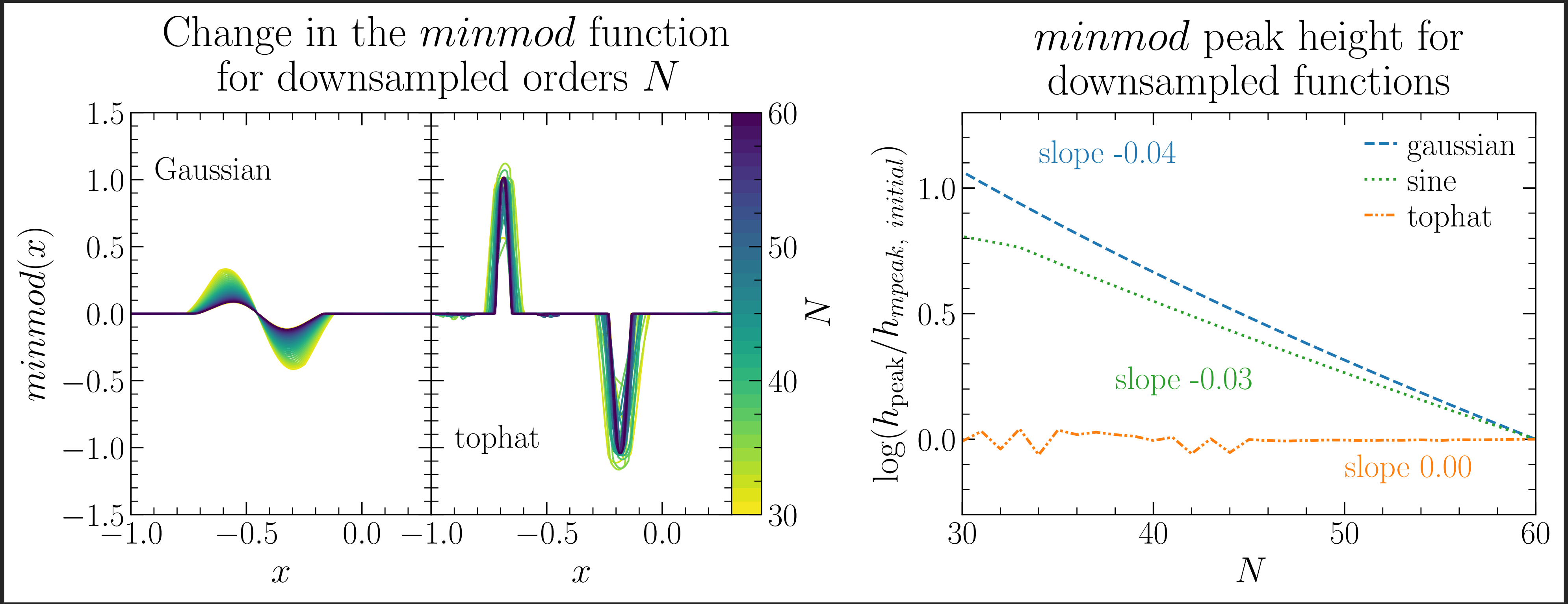


DYNAMICALLY EVOLVING DISCONTINUITIES
require

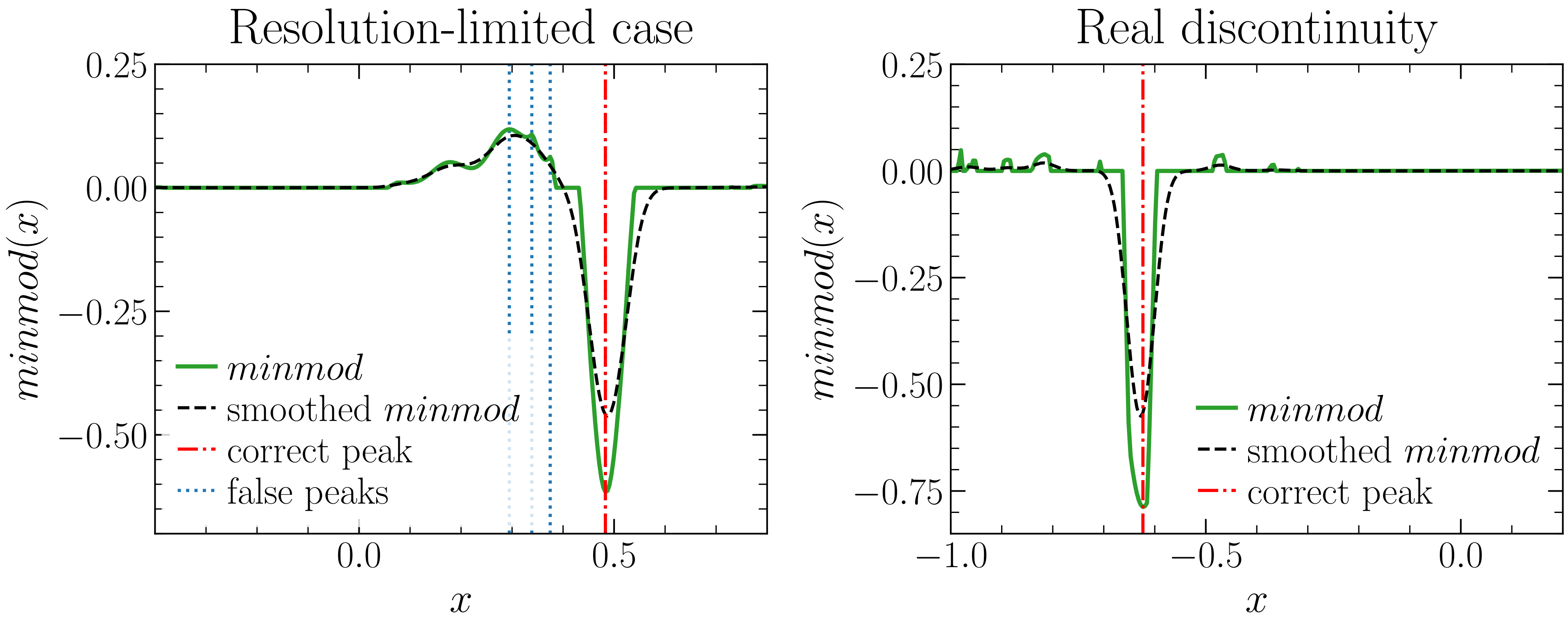
correct determination of
mollification onset

robust
edge detection

correct determination of **mollification onset**

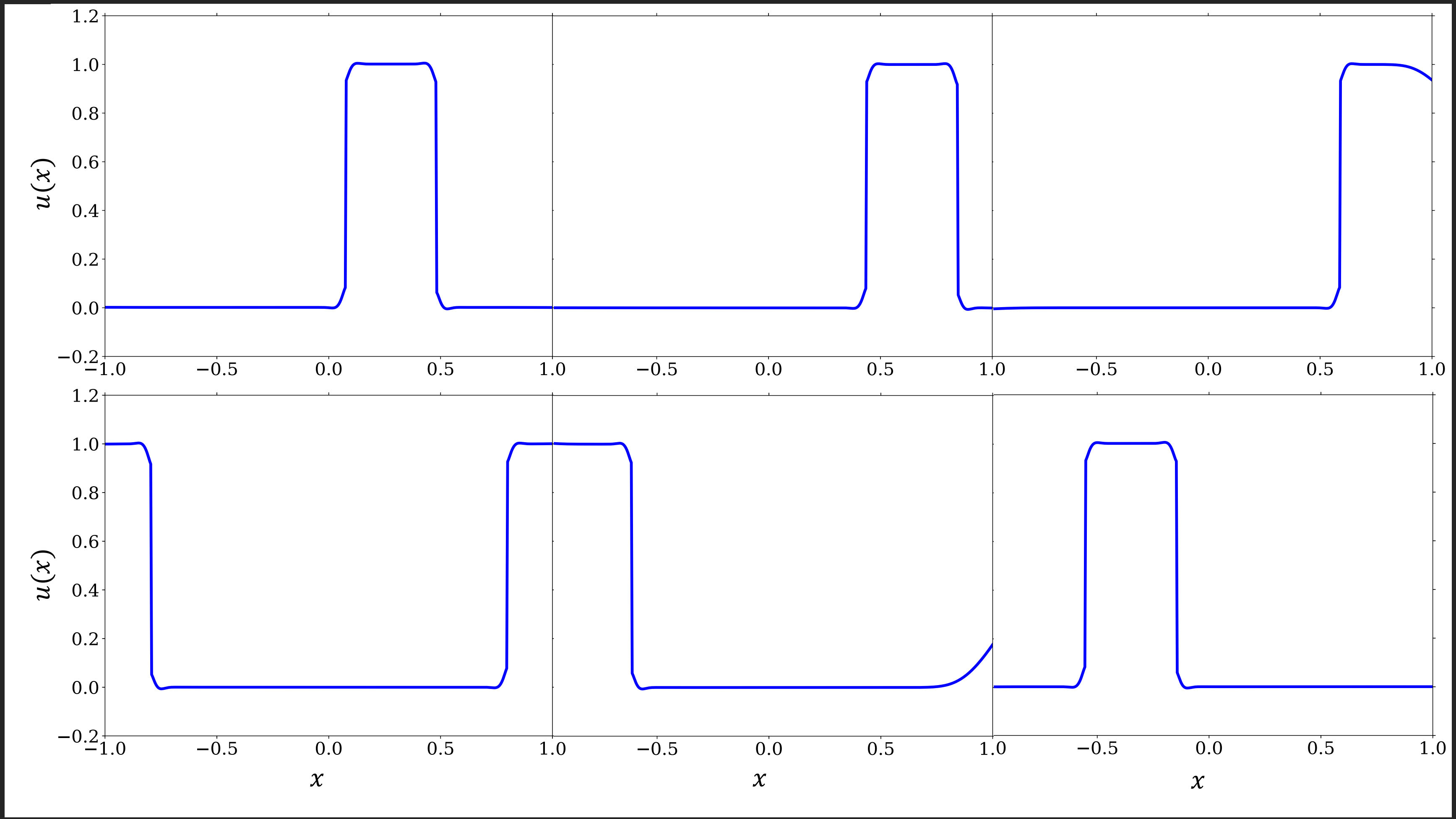


robust edge detection



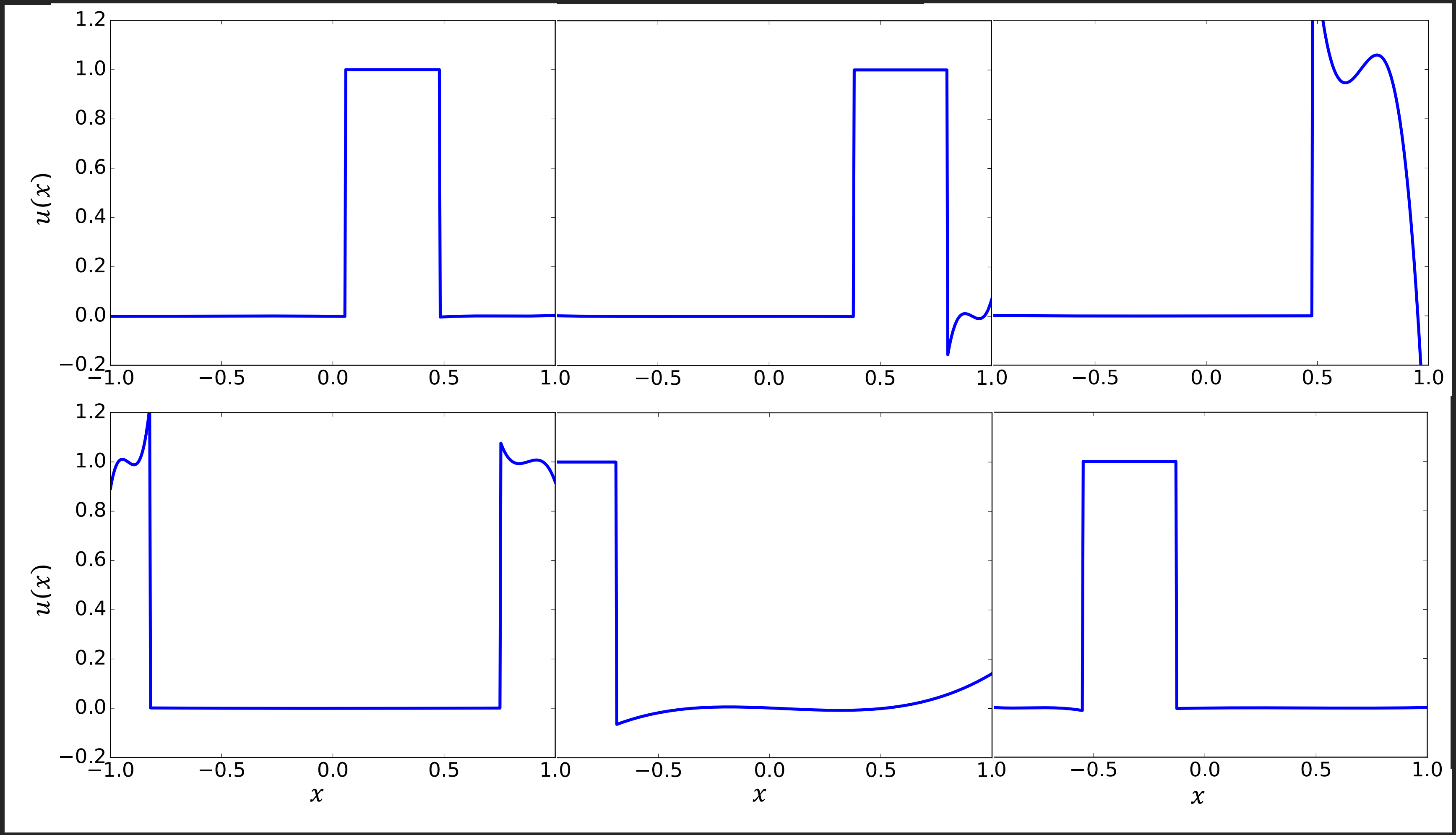
Toy example: 1D advection equation

$$\partial_t u + c\partial_x u = 0$$



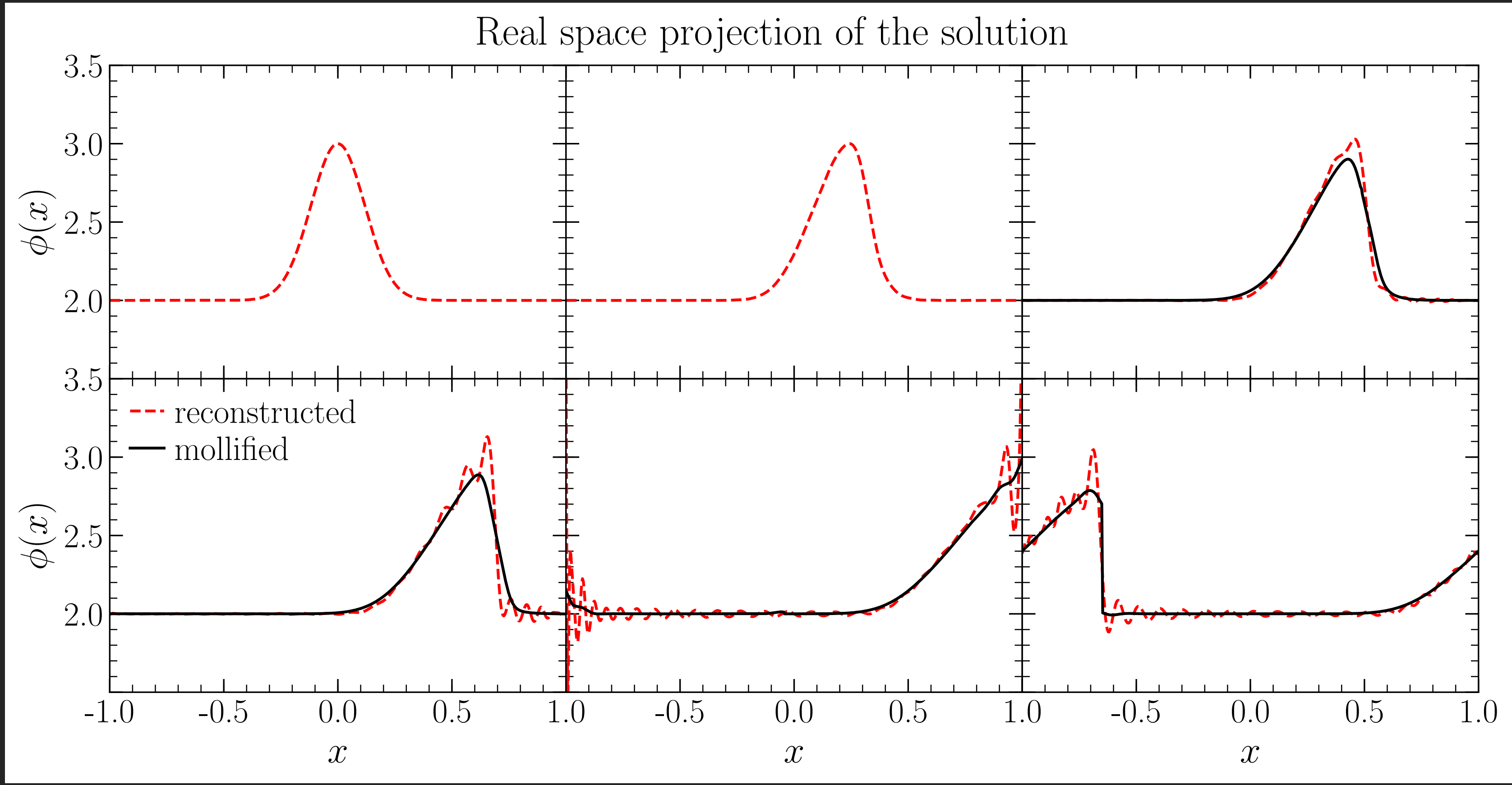
arXiv:1712.09952

comparison with the Gegenbauer reconstruction

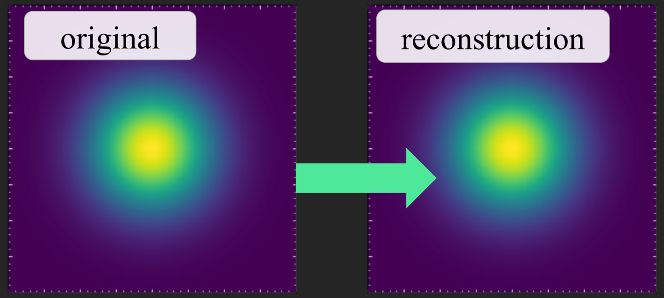


courtesy of
J. M. Miller

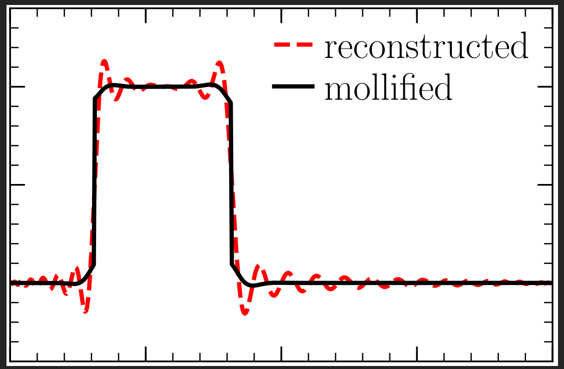
Advanced toy example: 1D inviscid Burgers' equation: $\partial_t \phi + \phi \partial_x \phi = 0$



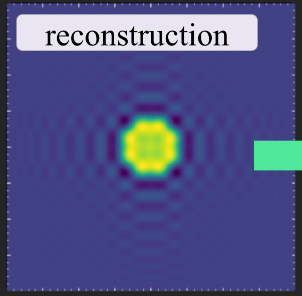
SUMMARY & OUTLOOK



- **pseudospectral methods** are a **highly accurate** means of solving **continuous** problems



- they suffer the **Gibbs' phenomenon** in presence of **discontinuities**
- the Gibbs phenomenon can be **robustly removed** in post-processing via **edge detection** and **mollification** (arXiv:1712.09952, Piotrowska et al. in prep.)



- taking advantage of the robustness of mollification, we are currently **extending our working framework to 2D**